**ENGINEERING ANALYSIS**

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**HOMEWORK #3 PROBLEM 2**

Consider the initial boundary value problem for the two dimensional heat equation:

a) Use separation of variables and the given boundary conditions to prove that

First the derivatives are:

Substitute them into the differential equation to get,

We can factor out the on the right hand side of the above equation:

By division of common terms we get,

Therefore, we have the following equations:

Let us examine the second differential equation: . Bring all terms to one side to get,

If we factor ψ we get,

Then isolate terms we get,

We get the following differential equations:

If we examine the first equation and let we get the following characteristic equation:

It has the following solutions for μ > 0:

This means that the solution for ψ will be of the form: . If we utilize the boundary conditions: . So at , this means that A = 0; hence, . If we consider the next part of the boundary conditions we get: . Therefore,

Therefore, the corresponding functions for ψ are:

Let us now consider the second equation for ϕ:

If we make the substitution: , we will get the following characteristic equation:

With roots: , so our solution to ϕ will have the form:

From the boundary conditions we get the following: . If we substitute x = 0 into ϕ we get: . Therefore, ϕ will be of the form:

If we utilize the second boundary condition and substitute x = L we get:

For the above case to be true we must have:

Hence:

But recall, , if we substitute we get:

And the corresponding functions of ϕ are:

So in summary we have the following relationships:

Let us now solve for the solution to the temporal equation . If we isolate the ‘h’ terms we get:

Hence,

But we already know λ:

We have:

So a solution to our heat equation is:

Part b)

Let us consider m = 1, then: , but remember for m = 1 we can also have every linear combination of this solution so:

For m = 2 we get:

For m = 3 we get:

We can continue this indefinitely and the sum of these solutions is also a solution to the heat equation:

Therefore,

Let us now consider the initial condition

Let us call the inner summation since it is independent of y:

Notice that this is a Fourier Sine series; hence,

But is the following:

Another Fourier Sine series expansion; thus,

But recall,

We can substitute this into to get,

Rearrange the integral a little and get,